PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH **19,** 020113 (2023)

Electromagnetic field presented in introductory physics textbooks and consequences for its teaching

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(Received 16 March 2023; accepted 29 June 2023; published 17 August 2023)

Textbooks play a fundamental role in teaching and learning in school science classrooms. In this paper, we investigate the presentation of the nature of the electromagnetic field in a dozen of the world's most popular introductory university physics textbooks. We analyze, from an epistemologically based teaching approach, the didactic treatment of the electromagnetic field in relation to its sources, Maxwell's laws and electromagnetic waves. With this objective, we elaborate a rational reconstruction of the developments that led to the formulation of the nature of the electromagnetic field, Maxwell's laws and their meaning, as well as electromagnetic waves. Next, we formulate criteria based on the key aspects derived from the reconstruction that are useful in the evaluation of electromagnetism textbooks at the introductory level and apply them to the sample of selected books. In light of the results, we reflect on their consequences for teaching. Our analysis indicates the existence of certain inconsistencies in the approach to the electromagnetic field and its relationship with its sources, Maxwell's laws and electromagnetic waves in many of the books analyzed.

DOI: 10.1103/PhysRevPhysEducRes.19.020113

I. INTRODUCTION

The teaching and learning of electromagnetism has been the object of multiple studies in Physics Education Research (PER) [1–7]. Nevertheless, it still raises significant challenges in many aspects, such as the attribution of causal relationships between electromagnetic fields and the difficulty in identifying the nature of their sources [8–14]. In fact, research into high school and university students' difficulties in understanding and implementing Maxwell's laws has systematically proven that a significant percentage of students still rely on misconceptions [15–20]. This poor learning progress has been attributed to different factors, including the nature of conventional science teaching and the difficulties that hinder better learning of scientific theories. Empirical research has also established that physics textbooks play a crucial role when it comes to presenting scientific theories and models in an accurate and coherent way. There is no doubt about the need to do further research into the representation, the logical correction, the structure, and the systematics of the contents in textbooks, regarding specific curriculum topics. In this paper, we analyze the ways the nature of the electromagnetic field is presented in a dozen textbooks, which rank among those most frequently used to teach introductory physics at universities.

The analysis of textbooks is justified by the importance of their influence on both teaching and learning. Research has systematically established that, to a large extent, science textbooks dictate the content and emphasis of

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science study plans, as well as the nature and scope of instructional activities and the discourse in most classrooms [21–23]. Undoubtedly, they are a major curricular and didactic resource for both junior and experienced science teachers. Students also use them in addition to other online resources to do their homework and prepare for exams [24]. Thus, the structure, logic, and coherence of the contents are crucial for its educational efficiency [25,26].

International organizations such as United Nations Educational, Scientific, and Cultural Organization promote the educational analysis of textbooks and put forward factors to consider in such analysis [27]. The American Association for the Advancement of Science (AAAS) Project 2061 defends the development of analysis protocols to evaluate the educational efficiency of science textbooks [28], due to the fact that they frequently represent over 75% of the tasks assigned by the teacher [29]. In this research, we focus on introductory physics textbooks published in the United States since they are the most widely used in introductory physics courses in science and engineering degrees worldwide. It can reasonably be supposed that the trends in American science textbooks are likely to significantly impact the teaching and learning of sciences in multiple world markets [30]. Furthermore, it is reasonable to suggest that textbook evaluation, based on criteria derived from the key aspects of rational reconstruction of classical electromagnetism, can provide teachers with an idea of how models or theories develop. Ignoring such key aspects and their development in textbooks can deprive students of the opportunity of becoming familiar with scientific practice and progress.

In accordance with the above considerations, our research question is the following: How do introductory physics textbooks explain the nature of the electromagnetic field? More specifically, we aim to answer these research questions:

- How do they explain Maxwell's laws and what attention is paid to the sources of the fields?
- How do they treat the generation and propagation of electromagnetic waves?
- How is the electromagnetic field presented in relation to different reference systems?

In order to answer these questions, we analyze, from an epistemologically grounded approach to teaching, how physics textbooks deal with the electromagnetic field in relation to its sources, Maxwell's laws and electromagnetic waves. To achieve this objective, first, we carry out a rational reconstruction of the developments that led to the formulation of the nature of the electromagnetic field, Maxwell's laws and their meaning, as well as electromagnetic waves in classical electromagnetism. Then, we formulate the criteria based on key aspects derived from the reconstruction, which may yield results in the evaluation of introductory-level electromagnetism textbooks. We examine the textbooks and, in the light of results obtained, consider their implications for teaching.

II. THEORETICAL FRAMEWORK

Recent studies that examine science teaching underpin the importance of both the history and the philosophy of sciences for science teaching [31,32]. The epistemologically grounded approach to teaching involves considering contributions related to the nature of science and its influence on teaching [33–36]. This approach indicates that knowledge results from a complex process where problems are solved and initial hypotheses tested, a process that enables the identification of the way ideas evolve to their current status [37]. Recognizing the evolution of the concepts underlying the different models can contribute to identifying epistemological and ontological barriers that scientists have to overcome in order to develop their theories. Such epistemic characteristics can usefully inform teaching approaches and help avoid inaccurate or excessively simplistic perspectives. The new science curricula suggest that students develop a deep understanding of the conceptual knowledge of science and the arguments associated with scientific practice that justify scientific theories (the epistemology of science) [38,39]. As a consequence, textbooks are required to not only focus on the "products" of science, i.e., concepts, theories, principles, and laws of nature but also put them into context within the epistemological characteristics that enabled their development [40].

The epistemologically grounded teaching approach about a specific topic needs to promote the dual goals: (a) to help students develop a scientific understanding of the concepts, laws, and models and (b) to safeguard against oversimplifications that clash with how the topic is conceptualized in the realm of physics [41]. Is in the second goal where the epistemological analysis is a useful instrument, defining the key elements of theory production requires an epistemological analysis of the content to be taught [35,42–44]. In this study, we understand epistemological analysis, the analysis of the historical developments of the theory taking into account the arguments of scientific practice that led to significant advances toward the current theory and that identifies the key elements of scientific knowledge of the theory to be taught, in this study, for an introductory physics level [34,45]. Through this gradual elaboration of the electromagnetic field as a framework for analyzing systems and interactions, certain fundamental epistemic features emerged, including, for example, its unifying and transphenomenological nature, and the applicability of the electromagnetic nature of waves. Defining these features could usefully inform attempts to design teaching and textbooks on this topic [42]. The epistemological analysis as an educational tool for teaching transposition provides a way for clearly identifying key science ideas that construct the conceptual content to be taught at a specific level [34]. In the following section, we will develop the epistemological analysis for the case of the nature of the electromagnetic field.

The history of electromagnetic theory from the 19th to the late 20th century shows the models of Faraday, Maxwell, Lorentz, and Jefimenko evolving in rapid succession. They had to overcome competing models based on ontological beliefs and rival research programs, such as the theory of action at a distance or the ether theory. This period, in which Maxwell's laws present a framework for analyzing electromagnetic interactions and identifying their sources, has spurred much debate and controversy in the history and philosophy of physics literature [46–49]. According to this perspective, it is important to analyze introductory university-level electromagnetism textbooks to determine the extent to which they address the key aspects of the classical theory of the electromagnetic field, its fundamental laws, and the sources of the electromagnetic field.

III. CONTRIBUTIONS OF EPISTEMOLOGY OF PHYSICS ON THE NATURE OF THE ELECTROMAGNETIC FIELD

During the 19th century, the confrontation between two major models—one based on action at a distance and the other on a theory of fields—resulted in a crucial advance of the electromagnetic theory [47]. Several factors influenced this process and the final result was the formation of the currently accepted, modern concept of the electromagnetic field. The first factor that affected the evolution of the conceptualization of electromagnetism was the contribution of Michael Faraday, who conceived a model of fields where interactions are transmitted as disturbances along the medium, as opposed to the previous ideas of instantaneous transmission at a distance [46]. This vision provided a reasonable interpretation of various electrical and magnetic interactions.

In the mid-19th century, Maxwell, inspired by the ideas of Faraday and William Thomson, developed a theory of the electromagnetic field based on the idea of continually transmitting electrical and magnetic actions. An essential component was the lines of force (introduced by Faraday in his model) as states of mechanic ether governed by Newton's laws [49]. In 1855, Maxwell published "*On Faraday's Lines of Force*," where he mathematically formulates the lines of force based on the use of analogies with the movement of an incompressible fluid [50]. An important landmark in the development of the theory is the conceptualization of ether as a quasimaterial element that supports the lines of force. Not surprisingly, in the following years, Maxwell's objective was to find a mechanical model for ether that could describe the electromagnetic field and allow to determine the speed of propagation of interactions. This process is completed in 1861, with the publication of "*On Physical Lines of Force*," where he takes his analogies to a different level by introducing an extremely complex mechanical model of ether that allows him to develop an electromagnetic theory of light and deduce the propagation speed of electromagnetic waves [48].

One of the most original aspects of Maxwell's theory was the introduction of electrical displacement and the current of displacement as another source of field. In 1864, Maxwell clearly described the meaning of both magnitudes.

> "Electrical displacement consists in the opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body […] The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity…" [51] (p. 554)

Years later, in *"A Treatise on Electricity and Magnetism,"* he makes clear his position on the displacement current as a source of magnetic fields:

> "The current produces magnetic phenomena in its neighborhood […] We have reason for believing that even when there is no proper conduction, but merely a variation of electric displacement, as in the glass of a Leyden jar during charge or discharge, the magnetic effect of the electric movement is precisely the same." [52] (pp. 144–145).

Given Maxwell's conception of space, and more specifically his conviction about the existence of ether, the displacement current, conceived as a consequence of the variation of the electric displacement in any mechanical medium, was always associated with a motion of bound charges.

The second important factor that influenced the development of the electromagnetic theory was the progressive rejection of the ether theory and the beginning of the adoption of an analytic interpretation. Maxwell, aware of the limitations and difficulties associated with his mechanical model of ether, decided to make it independent of the electromagnetic field. In 1864, he published "*A Dynamical Theory of the Electromagnetic Field*," where he presented eight equations of the electromagnetic field and an electromagnetic theory of light that can be experimentally contrasted [46]. In this task, he faced the difficulty of how to interpret the set of developed equations. This is when the analytical interpretation arises: the charge and the currents become fundamental magnitudes and the field acts directly on the matter in the point of interest, and as a consequence, the mechanisms of continuous action imagined by Faraday disappear [49]. Nine years later, Maxwell published his most important work, "*A Treatise on Electricity and Magnetism*," where he presented the whole electromagnetic theory in detail. It continues with his analytical interpretation of the model, although he still firmly believed in the existence of some mechanism subordinated to Newton's laws [49].

Despite Maxwell's formidable advances, there were still many things that needed to be explained. These were, among others, the development of an adequate theory of charge, the explanation of the properties of the dielectrics, and the complete determination of the fields around time-varying charges and currents. In addition, experiments needed to be conducted to provide evidence that would support his field theory [47]. Years after Maxwell's death in 1887, Heinrich Hertz experimentally demonstrated the existence of electromagnetic waves, thus confirming the field theory as an alternative to the theory of action at a distance [46]. At the same time, Oliver Heaviside, used vector calculus to analytically express the equations of Maxwell's fields, writing the equations as we know them today [49].

In 1892, Hendrik Lorentz took one more step in the development of the classical electromagnetic field theory by assuming that all charged bodies have charged particles and that ether is immobile and unperturbed by their movement. This, consequently, means that the only possible interpretation of the field equations is the analytical one [49]. Lorentz further assumed that electric and magnetic fields are qualitatively different, although they are produced and propagated outward by the charged particles [53]. In contrast to Maxwell's view, Lorentz considered an immobile ether and allowed for the existence of a displacement current in the absence of matter, which would make it a simple term directly proportional to the rate of change in the electric field [54].

The third important factor in the development of field theory was the contributions that identified electric and magnetic fields as related entities, the definitive rejection of the existence of ether and the clarification of the sources of the fields. In 1905, Albert Einstein postulated the theory of special relativity. In this framework, he shows that Maxwell's laws adopt the same expressions in all inertial reference frames and that the components of the electric and magnetic fields relate to each other in the various inertial reference frames by means of Lorentz's transformations. He concluded that contrary to Maxwell's and Lorentz's theories, electric and magnetic fields should not be considered separate entities but are in fact both part of one entity: the electromagnetic field [48]. In this way, Einstein managed to resolve the asymmetries that appeared when applying Maxwell's electrodynamics to moving bodies [55]. If an observer detects only a magnetic field in a reference frame, another observer in motion will always measure an electric and a magnetic field. Similarly, if an electric field is measured in a reference frame, an electric and a magnetic field will always be detected in another. From Einstein's theory of special relativity onwards, the existence of ether lost its *raison d'être*, and by the 1920s, its existence in the scientific community was a thing of the past, and the analytic interpretation of the field equations became the dominant one.

Another important milestone came in the 1960s when Oleg Jefimenko elucidated the problem of the sources of the electromagnetic field [56]. In his work, he presents general solutions for the electric and magnetic fields at a given instant and point in space as a function of charge and current distributions which can be expressed as Ref. [57]

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int\left(\frac{\rho(r',t')}{|\mathbf{r}-\mathbf{r}'|^3} + \frac{\dot{\rho}(r',t')}{c|\mathbf{r}-\mathbf{r}'|^2}\right)(\mathbf{r}-\mathbf{r}')dv' - \frac{1}{4\pi\varepsilon_0}\int\frac{\dot{\mathbf{J}}(r',t')}{c^2|\mathbf{r}-\mathbf{r}'|}dv', \tag{1}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int\left(\frac{\mathbf{J}(r',t')}{|\mathbf{r}-\mathbf{r}'|^3} + \frac{\dot{\mathbf{J}}(r',t')}{c|\mathbf{r}-\mathbf{r}'|^2}\right)\times(\mathbf{r}-\mathbf{r}')dv', \tag{2}$$

where the fields $\mathbf{E}$ and $\mathbf{B}$ are evaluated at position $\mathbf{r}$ at time instant t, being $\mathbf{r}'$ the distance from the origin of coordinates to charge density ρ and current density $\mathbf{J}$, c is the speed of light, and $t' = t - |\mathbf{r}-\mathbf{r}'|/c$.

From these equations, Jefimenko concludes that the sources of the fields are charge and current distributions [8,9,12,58–60]. Given that electric and magnetic fields have the same sources, when charge and current distributions vary over time, both fields are generated simultaneously and correlated through Faraday's and Ampere-Maxwell laws [12]. All terms of Maxwell's laws in differential form are evaluated at the same instant of time and at the same point in space, therefore, they do not imply cause-and-effect relationships and none of them can be the source of the other [10,11]. From the point of view of the relativistic formulation of electromagnetism, electric and magnetic fields form a single entity, the electromagnetic field tensor [61]. Just as it does not make sense to think that one component of a vector is the cause of another component of the same vector, it does not make sense that one component of the tensor is the cause of another component of the same tensor [9]. Hence, it is impossible to obtain the temporal evolution of one of the fields knowing the temporal evaluation of the other since it ignores that they are components of the same entity and, therefore, cannot interact with each other.

In summary, the historical and epistemological development of electromagnetic field theory reveals important milestones in the process: (a) a mechanistic interpretation of electromagnetic interaction (Faraday-Maxwell); (b) a further development that lead to an analytical interpretation based on Maxwell's laws which contradicts the earlier model (Lorentz, Heaviside) and succeeds in experimentally contrasting its prediction of electromagnetic waves (Hertz); and (c) an evolution toward a unified theory of the electromagnetic field, with Lorentz's transforms (Einstein) and the definition of charge and current distributions as the only sources of the electromagnetic field (Jefimenko). Through this gradual elaboration of the theory with Maxwell's laws as a framework which helped us analyze electromagnetic interactions, certain fundamental epistemic features arise that allow us to define a number of key concepts to be accounted for in the textbooks, from an epistemologically grounded approach to teaching. Next, we define these key ideas in relation to the classical electromagnetic field theory, the equations that govern it and the sources of the fields for university-level introductory physics:

- KC1. The interpretation of Maxwell's laws implies noncausal relationships between the fundamental magnitudes of electromagnetic theory ($\mathbf{E}$, $\mathbf{B}$, $\mathbf{J}$, and ρ) at the same instant of time.
- KC2. The sources of electromagnetic fields are charge and current distributions.
- KC3. The cause of the fields in an electromagnetic wave (as well as that of electric and magnetic fields) at a certain instant is the distribution of charges and currents (generally time varying) at an earlier time.
- KC4. The electric and magnetic fields associate and form one single entity, the electromagnetic field, whose components are linked in different inertial systems by means of Lorentz's transformations.

IV. METHODOLOGY

A. Selection of textbooks and assumptions of the analysis

In the sample of selected textbooks, we use the most appropriate nonprobabilistic strategy, known as "purposive sampling" according to Cohen *et al.* [62]. Our aim is for the sample to be representative, i.e., to reflect a situation common to the research objective. To select the sample, we analyzed the bibliography used in the introductory courses of electricity and magnetism of the physics and engineering degree courses of the most outstanding universities in Spain, Latin America, and the United States according to the SCImago ranking [63]. In the selection, we imposed the condition of having been published in the last 15 years in order to incorporate the abundant research in PER related to electromagnetism and Maxwell's laws. This condition left out of the sample some textbooks that were very influential in introductory physics programs in the past. From the study of the bibliographies analyzed, we found that a set of seven textbooks (the first seven in Table I) cover almost all of the courses studied. In order to broaden the scope of the investigation, we deemed it convenient to incorporate into the analysis some books of a similar level that were published recently, as well as the text *Six Ideas That Shaped Physics* by Thomas Moore, which is characterized by paying special attention to the results emanating from PER. In each textbook, we analyze the chapters dealing with magnetic fields, Faraday's and Ampere-Maxwell's laws, ac circuits, as well as with electromagnetic waves and special relativity. In Table I, we show the name of each textbook of the sample and the chapters analyzed.

The analysis was carried out by the four authors of the article who are professors or assistant professors in physics departments. They have extensive experience in teaching introductory physics courses, as well as courses in electrodynamics, thermodynamics, and computational physics [64].

We have taken into account that the epistemological criteria defining the key concepts involve abstract concepts whose mathematical developments, such as the Jefimenko equations, are dealt with in electrodynamics courses. However, in the analysis, we have restricted ourselves to the introductory level of the textbooks analyzed. In general,

TABLE I. Textbooks and chapters analyzed.

Authors	Titles	Year	Chapters analyzed
P. A. Tipler and G. Mosca	Physics for scientists and engineers	2008	26 to 30 and 39
J. Walker, R. Resnick, and D. Halliday	Fundamentals of physics	2014	28 to 33 and 37
D. C. Giancoli	Physics for scientists and engineers with modern physics	2014	27 to 31 and 36
R. W. Chabay and B. A. Sherwood	Matter and interactions	2015	17 and 20 to 23
R. A. Serway and J. W. Jewett	Physics for scientists and engineers	2019	28 to 33 and 38
H. D. Young and R. A. Freedman	University physics with modern physics	2020	27 to 32 and 37
R. Knight	Physics for scientists and engineers: strategic approach with modern physics	2022	29 to 32 and 36
W. Bauer and G. Westfall	University physics with modern physics	2014	27 to 31 and 35
D. M. Katz	Physics for scientists and engineers: Foundations and connections	2015	30 to 34 and 39
T. A. Moore	Six ideas that shaped physics: Electric and magnetic fields are unified	2017	8 to 17 and appendix A
R. L. Hawkes, J. Iqbal, F. Mansour, M. Milner-Bolotin, and P. J. Williams	Physics for scientists and engineers: An interactive approach	2019	24 to 27 and 30
R. Wolfson	Essential university physics	2020	26 to 29 and 33

the approaches used are based on qualitative reasoning inspired by experimental facts rather than on mathematical formalism typical of advanced courses. So, we have analyzed the key ideas as deeply as the textbooks themselves define them.

We designed a textbook-review protocol based on three criteria from the epistemological analysis and the key ideas defined in the previous section. We consider the theoretical explanations as well as the questions and solved examples used to illustrate the explanations. We have included the solved examples in the analysis because they illustrate the explanations of the theory and have allowed us to clarify the meaning that the authors explained in the theory sections. For the same reason, the analysis does not include the questions and exercises proposed at the end of the chapters. Each criterion is developed through a protocol that constitutes a tool for analysis, which is explained in the following section.

B. Instrument

We describe in this section the instruments of analysis for each criterium.

Criterion 1. The treatment of Faraday and Ampère-Maxwell's laws in relation to the sources of the electric and magnetic field.

Maxwell's laws do not imply cause-and-effect relationships (KC1) but present mathematical relationships between different magnitudes at the same instant of time. There cannot be a causal relationship between the different terms of the equations. We understand causality according to the principle of delayed action:

> "there is always a time delay between the cause and the effect, the former being prior in time to the latter so that (relatively to a given physical system, such as a reference system), C and E cannot be both distant in space and simultaneous." [65] (p. 62)

Thus, charge and current distribution are the causes of the fields which determine line integrals based on Faraday and Ampère-Maxwell laws (KC2).

Faraday's Law in integral form

$$\varepsilon = \oint \mathbf{E} \cdot \mathbf{dl} = -\frac{d}{dt} \int \mathbf{B} \cdot \mathbf{dA}, \tag{3}$$

Ampère-Maxwell's law in integral form

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot \mathbf{dA}. \tag{4}$$

Using criterion 1, we will examine whether the following aspects are explained in the textbooks:

C1.1 Faraday and Ampère-Maxwell's laws describe mathematical relationships between different magnitudes at the same instant of time.

C1.2 It is explained that the fields described in Faraday and Ampère-Maxwell's laws do not imply causal relationships between electric or magnetic field operators and the generation of electric or magnetic field.

Criterion 2. Presenting the generation and propagation of electromagnetic waves in relation to their sources.

Changes in the characteristics of an electromagnetic wave's fields at a certain point are associated with changes in the distribution of charges or currents that generated it at an earlier time (KC3). This aspect of the electromagnetic theory is key to understanding that information propagates with finite speed and there exists a causal relationship between the wave and its sources. Therefore, this criterion will examine whether the following aspects are explained in the textbooks:

C2.1 Time-varying charge or current distributions over time generate electromagnetic waves, linking the electromagnetic wave to its sources.

C2.2 Electric and magnetic fields in electromagnetic waves are associated with each other and are simultaneous in time without establishing a causal relationship between them.

Criterion 3. Presenting the electromagnetic field as a single entity in relation to different reference systems.

The analysis of the electromagnetic field in different reference systems related by Lorentz transformations allows us to explain clearly that there is one single electromagnetic field that presents different components, in terms of **E** and **B**, to different observers (KC4). In this respect, this criterion will analyze whether the following aspects are included in the textbooks:

C3.1 It is explained that, if an electric or magnetic field is observed in one inertial reference frame, both fields are detected in any other reference frame.

C3.2 Lorentz transformations for electric and magnetic fields are introduced, the electromagnetic field is presented as a single entity with two components: the electric field and the magnetic field.

C. Data analysis

In the first phase of data analysis, one of the researchers reviewed six books from the sample according to the initial protocol that included the three criteria. Subsequently, all four researchers met to analyze and discuss the analysis protocol and suggest improvements on the type of information expected. The analysis adopted does not involve comparing a given textbook with an ideal book or ranking it in relation to the criteria. In general, in each textbook, we aim to identify explanations, examples, or questions that provide evidence about the presentation of the topic in relation to the criteria established, which we agreed to evaluate according to the following classification:

- Complies (C): the treatment of the theory, law, or concept is considered appropriate to the criterion set.

- Partially complies (PC): some aspects of the criterion are mentioned but not explained. When the content of a criterion is not explicitly detected in the text but seems to be commented upon, it is included in this category.
- Does not comply (NC): mentions elements that are not compatible with the theoretical framework or presents information that could lead to misunderstandings.
- Does not address the issue (NT): does not address the issues included in the criterion.

The revised protocol was applied by two of the researchers who analyzed the 12 textbooks independently. Subsequently, all researchers met to discuss the results obtained and to clarify any doubts that arose during the analysis process. We agreed that the four researchers will carry out a second round of reviews, with the final protocol (see Results below), to guarantee the validity of the analysis. The review and validation protocols, as well as the two rounds of review in which each textbook has been analyzed by at least three researchers, allow us to reduce possible bias in the information extraction of this literature review. At the end of the second round, there was a strong consensus among the researchers on whether or not the criteria were met and on the comments on each textbook. Finally, we determined Cohen's kappa coefficient, for the categories defined by the four evaluators. The statistic expresses a measure of the degree of agreement between evaluators who classify items into mutually exclusive categories and takes into account the possibility that they may agree by chance. We obtained an overall value of 0.94. Values above 0.80 imply a high degree of agreement [66]. In the next section, we present the results of Cohen's kappa coefficient for each particular criterion.

V. RESULTS

Here we describe the results obtained from the evaluation of each criterion. Summarized results for all criteria can be found in Table V.

A. Textbooks treatment of Maxwell's laws in relation to electromagnetic field sources

Criterion 1 assesses the ability to identify the sources of the fields and to recognize that Maxwell's laws describe mathematical relationships between different terms at the same instant of time. The Cohen's kappa coefficient obtained for this criterion was 1.0. Table II shows the results of the evaluation of the aspects included in criterion 1 for each of the textbooks.

As can be seen from Table II, all texts appropriately explain that Faraday and Ampère-Maxwell's laws describe mathematical relationships between different magnitudes at the same instant of time, giving correct analytical descriptions of the laws (criterion 1.1). For example, Tipler and Mosca [67] (p. 1034) state that

TABLE II. Results corresponding to treatment of Faraday and Ampère-Maxwell's laws in relation to the sources of the electric and magnetic field. C1.1 refers to the explanation that these laws describe mathematical relationships between different magnitudes at the same instant of time while C1.2 implies that they do not imply causal relationships between the fields. In this table and the followings, the keys are C, complies; PC, partially complies; NC, does not comply; NT, does not address.

Authors	Criterion 1.1	Criterion 1.2
P. A. Tipler and G. Mosca	C	NC
J. Walker, R. Resnick, and D. Halliday	C	NC
D. C. Giancoli	C	NC
R. W. Chabay and B. A. Sherwood	C	C
R. A. Serway and J. W. Jewett	C	NC
H. D. Young and R. A. Freedman	C	NC
R. Knight	C	NC
W. Bauer and G. Westfall	C	NC
D. M. Katz	C	NC
T. A. Moore	C	C
R. L. Hawkes, J. Iqbal, F. Mansour, M. Milner-Bolotin, and P. J. Williams	C	NC
R. Wolfson	C	NC

> "Faraday's law states that the line integral of the electric field E around any closed curve C equals the negative of the rate of change of the flux of the magnetic field B through any surface S bounded by curve C."

Whereas Ampère-Maxwell law [67] (p. 1034)

> "…states that the line integral of the magnetic field B around any closed curve C equals 0 multiplied by the sum of the current I through any surface S bounded by the curve and the displacement current I_D through the same surface."

It is worth pointing out that the books analyzed complement the analytical descriptions of Maxwell's laws with different application examples where it is clear that the different magnitudes involved are evaluated at the same instant of time. For example, Bauer [68] (p. 880) presents the following problem of Faraday's law:

> "A current of 600 mA is flowing in an ideal solenoid, resulting in a magnetic field of 0.025T inside the solenoid. Then the current increases with time, t, according to
>
> $$i(t) = i_0[1 + (2.4s^{-2})t^2]$$
>
> If a circular coil of radius 3.4 cm with $N = 200$ windings is located inside the solenoid with its normal vector parallel to the magnetic field (Fig. 29.8), what is the induced potential difference in the coil at $t = 2.0s$?"

In solving this problem, an expression is found for the magnetic field generated by the solenoid as a function of the current intensity. Then, to determine the induced emf, the magnetic field flux is calculated and derived with respect to time, resulting in

$$\varepsilon = -AB_0[2(2.4s^{-2})t], \tag{5}$$

where A is the cross-sectional area of the solenoid, while B_0 is its magnetic field in $t = 0$ s. Finally, the induced emf at $t = 2.0$ s is found. The presentation of problems like the one described, where the emf has to be determined for a certain value of t, allows us to infer that the terms involved are evaluated at the same instant of time.

According to criterion 1.2, the fields described in Faraday and Ampère-Maxwell's laws do not imply causal relationships between electric or magnetic field operators and the generation of electric or magnetic field. This aspect of Maxwell's laws is only taken into account in two of the textbooks analyzed. One of the texts that complies with the criterion is Moore's [69] (p. 271) where, explaining Faraday's law, he explicitly states that

> "I have been very careful to state that this is the electric field that is correlated with the changing magnetic field, not created by that field. Electromagnetic fields are created only by stationary or moving charged particles."

However, 10 of the 12 textbooks analyzed interpret Faraday and Ampère-Maxwell's laws as describing cause-and-effect relationships between different terms, such that the time-varying magnetic field is the source of the electric field (Faraday's law) and vice versa, the time-varying electric field is the source of the magnetic field (Ampère-Maxwell's law). This interpretation is explicitly presented, for example, when summarizing the physical meaning of Maxwell's laws. It states:

> "Faraday's law: An electric field can also be created by a changing magnetic field.
> Ampère-Maxwell law, first half: Currents create a magnetic field.
> Ampère-Maxwell law, second half: A magnetic field can also be created by a changing electric field." [70] (p. 939)

Interpretations like this are reinforced through the analysis of different situations where Faraday and Ampère-Maxwell's laws are applied. One of the most representative cases is the calculation of the electric field around a solenoid where the current intensity changes with time. For example, the following problem is entitled "E produced by changing B":

> "A magnetic field B between the pole faces of an electromagnet is nearly uniform at any instant over a circular area of radius as shown in Figs. 27(a) and (b). The current in the windings of the electromagnet is increasing in time so that changes in time at a constant rate dB/dt at each point. Beyond the circular region $(r > r_0)$ we assume $B = 0$ at all times. Determine the electric field E at any point P a distance r from the center of the circular area due to the changing B." [71] (p. 892)

In this problem, the time-varying magnetic field is explicitly described as the cause of the electric field, however, both fields are generated at the same time and have a common cause: the varying current.

In the case of Ampère-Maxwell law, the ten textbooks that do not comply with criterion 1.2 provide explanations of the effects of the displacement current that could be a source of misunderstandings, for example:

> "Although changing electric flux isn't the same thing as electric current, it has the same effect in producing a magnetic field. For this reason Maxwell called the term $\varepsilon_0 d\Phi_E/dt$ the displacement current. The word displacement has historical roots that don't provide much physical insight. But current is meaningful because displacement current is indistinguishable from real current in producing magnetic fields." [72] (p. 582)

Assuming that the displacement current generates a magnetic field may lead students to believe that there is a cause-and-effect relationship between the different terms of Ampère-Maxwell law. Even more so, if we take into account that one of the first applications of the displacement current is the calculation of the magnetic field between the plates of a capacitor that is charging. For example, in one of the textbooks analyzed, the following situation arises:

> "You might well ask at this point whether displacement current has any real physical significance or whether it is just a ruse to satisfy Ampère's law and Kirchhoff's junction rule. Here's a fundamental experiment that helps to answer that question. We take a plane circular area between the capacitor plates (Fig. 29.23). If displacement current really plays the role in Ampère's law that we have claimed, then there ought to be a magnetic field in the region between the plates while the capacitor is charging." [73] (p. 972)

Next, the magnetic field between the capacitor plates is determined by the Ampère-Maxwell law. To do this, a closed curve placed between the capacitor plates is taken and the displacement current through the surface delimited by this curve is determined. Then, the displacement current is expressed as a function of the conduction current I, and the magnetic field is determined at a distance r from the symmetry axis of the capacitor, for points such that $r \leq R$, being R the radius of the plates, we obtain:

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}. \tag{6}$$

In arriving at Eq. (6), it is concluded that

> "When we measure the magnetic field in this region, we find that it really is there and that it behaves just as Eq. (29.17) predicts. This confirms directly the role of displacement current as a source of magnetic field. It is now established beyond reasonable doubt that Maxwell's displacement current, far from being just an artifice, is a fundamental fact of Nature." [73] (p. 972)

In contrast, in the textbooks that comply with the criterion, they do not associate the term $\varepsilon_0 d\Phi_E/dt$ with a current. For example, when analyzing the different terms of Ampere-Maxwell's law, Chabay and Sherwood [74] (p. 941) conclude that

> "We interpret the Ampere–Maxwell law as saying that a time-varying electric field is always associated with a magnetic field."

The fact that Ampere-Maxwell law can be used to calculate a magnetic field whose circulation is directly proportional to the speed with which the electric field flux varies does not mean that it is generated by it. As discussed in Sec. III, a time-varying electric field does not generate a magnetic field; the sources of the magnetic field between the plates of a charging capacitor are the surface currents in its plates and the conduction currents in the wires [8,75–77].

The results indicate that, depending on the context, ten textbooks recognize the presence of four possible sources of fields **E** and **B**: (a) charges; (b) current; (c) variation of **E** generates **B**; (d) variation of **B** generates non-Coulombic **E**. They do not consider it contradictory to indicate charges and currents as sources on the one hand and time-varying electric and magnetic fields on the other.

B. Presenting the generation and propagation of electromagnetic waves in relation to their sources

The study of the generation and propagation of electromagnetic waves provides a unique opportunity to clarify the sources of the fields, to recognize that information does not propagate instantaneously and that electric and magnetic fields in waves are correlated but there is no causal relationship between them. These aspects are assessed through criterion 2. The Cohen's kappa coefficient obtained for this criterion was 0.88. Table III shows the results of the evaluation of the aspects included in criterion 2 for each of the textbooks.

TABLE III. Summary of the results about the generation and propagation of electromagnetic waves in relation to the sources of the electromagnetic waves. C2.1 implies the explanation that time-varying charge or current distributions generate electromagnetic waves while C2.2 implies in electromagnetic waves both fields are associated with each other and are simultaneous in time without establishing a causal relationship between them.

Authors	Criterion 2.1	Criterion 2.2
P. A. Tipler and G. Mosca	C	C
J. Walker, R. Resnick, and D. Halliday	C	NC
D. C. Giancoli	C	NC
R. W. Chabay and B. A. Sherwood	C	C
R. A. Serway and J. W. Jewett	C	NC
H. D. Young and R. A. Freedman	C	NC
R. Knight	NC	NC
W. Bauer and G. Westfall	C	C
D. M. Katz	NC	NC
T. A. Moore	C	C
R. L. Hawkes, J. Iqbal, F. Mansour, M. Milner-Bolotin, and P. J. Williams	C	NC
R. Wolfson	C	NC

Table III reveals that a large majority of the textbooks analyzed explicitly describe that the sources of the waves are the accelerated charges. They do this using different approaches and depths, often relying on the analysis of radiation emitted by oscillating charges in an antenna.

Especially noteworthy is the textbook by Hawkes *et al.* [78], which takes a different approach based on the analysis of the fields generated by a point charge in different inertial reference frames. A point charge performing a uniform linear motion (URM) generates an electric field and a magnetic field around itself. An observer at rest with respect to the charge detects only a static electric field, so for such an observer, the charge does not radiate electromagnetic waves. With this in mind, Hawkes *et al.* [78] (p. 974) state

> "The laws of physics must be the same in all frames that move with a uniform velocity with respect to each other. So, if a charge cannot emit electromagnetic radiation in its own reference frame, it cannot emit electromagnetic radiation in any frame moving with uniform velocity with respect to the charge. Thus, a charge can emit electromagnetic radiation in free space only when accelerating."

Regardless of the approach used, we can see a general agreement on the fact that the electromagnetic wave consists of a disturbance of the propagating electromagnetic field and that the origins of such disturbances are accelerated charges. For instance, Wolfson [72] (p. 593) writes

> "All it takes to produce an electromagnetic wave is a changing electric or magnetic field…Ultimately, changing fields of both types result when we alter the motion of electric charge. Therefore, accelerated charge is the source of electromagnetic waves."

However, when electromagnetic wave generation processes are explained, we must pay attention to the language if we wish to avoid possible misunderstandings. In this respect, two of the textbooks analyzed do not comply with criterion 2.1. Depending on the context, they suggest two possible sources of electromagnetic waves: accelerated charges or a time-varying electric field. For example, when explaining the generation of the waves, one of the textbooks describes the transmitter of Hertz's original experiment, which consists of a voltage source connected to an induction coil using a switch and a parallel connected capacitor made of two metal rods separated by a gap and two spheres connected at their ends. It arrives to the following conclusion:

> "When the transmitter is turned on, a strong electric field builds up in its gap. The electric field ionizes the air, and acceleration of the resulting free electrons causes more ionization until the air conducts a spark. The plates of the capacitor charge and discharge periodically, while the sparks in the gap oscillate at the natural frequency of the LC circuit. The electric field oscillation creates an electromagnetic wave." [79] (p. 1094)

In our analysis of the way the textbooks address the relationship between electric and magnetic fields in electromagnetic waves, we found that every case demonstrates that in a plane wave the fields are in phase and are mathematically linked according to the equation:

$$\frac{E(t)}{B(t)} = c. \tag{7}$$

Which involves, according to Serway and Jewett [80] (p. 881),

> "…at every instant, the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light."

In contrast, when explaining the propagation of electromagnetic waves, only one-third of the selected textbooks explain propagation without establishing causal relationships between electric and magnetic fields (criterion 2.2). For instance, Bauer [68] (p. 943) writes that

> "Electromagnetic waves consist of electric and magnetic fields, can travel through vacuum without any supporting medium, and do not involve moving charges or currents. The existence of electromagnetic waves was first demonstrated in 1888 by the German physicist Heinrich Hertz (1857–1894). Hertz used an RLC circuit that induced a current in an inductor that drove a spark gap. A spark gap consists of two electrodes that, when a potential difference is applied across them, produce a spark by exciting the gas between the electrodes. Hertz placed a loop and a small spark gap several meters apart. He observed that sparks were induced in the remote loop in a pattern that correlated with the electromagnetic oscillations in the primary RLC circuit. Thus, electromagnetic waves were able to travel through space without any medium to support them."

We note that the author describes the propagation of waves in a way that is simple and understandable for students without referring to any interaction between the electric and magnetic fields of the electromagnetic wave.

In line with this, Chabay and Sherwood [[74], p 947] state that

> "Maxwell concluded that light must be a combination of time-varying electric and magnetic fields that can propagate through otherwise empty space, far from any charges or currents."

Later, when referring to the electric and magnetic fields of the electromagnetic wave, Chabay and Sherwood [74] (p. 948) write that

> "…a pulse of radiation must contain both electric and magnetic fields, so that the pulse can move forever through otherwise empty space, far from any charges. The transverse electric field is accompanied by a transverse magnetic field perpendicular to the electric field, with $\mathbf{E} \times \mathbf{B}$ in the direction of propagation."

It can be deduced from the explanation that the electric and magnetic fields in electromagnetic waves do not interact with each other, being generated at the same time. The explanation given by Chabay and Sherwood is in accordance with Eq. (7), according to which the fields in a point in space always vary in the same way and simultaneously so that one field cannot be the cause of the other.

However, two-thirds of the textbooks analyzed consider that the propagation of electromagnetic waves lies in a supposedly self-sustaining mechanism, originated by a time-varying electric field that is supposed to generate a time-varying magnetic field that would in turn generate an electric field and so on. Let us consider the following examples:

> "Once an electromagnetic wave is generated, it propagates on its own and does not require any physical medium to continue its propagation. The wave generates itself because a time-varying electric field generates a time-varying magnetic field, which in turn generates a time-varying electric field. This process is continually repeated." [78] (p. 962)
>
> "First consider the magnetic field. Because it varies sinusoidally, it induces (via Faraday's law of induction) a perpendicular electric field that also varies sinusoidally. However, because that electric field is varying sinusoidally, it induces (via Maxwell's law of induction) a perpendicular magnetic field that also varies sinusoidally. And so on. The two fields continuously create each other via induction, and the resulting sinusoidal variations in the fields travel as a wave—the electromagnetic wave." [81] (p. 976)

C. Presenting the electromagnetic field as a single entity in relation to different reference systems

The special theory of relativity laid the foundation for a new conception of electric and magnetic fields by no longer considering them as separate entities but as parts of a single entity, the electromagnetic field, which adopts different expressions depending on the reference system. These aspects of the electromagnetic field are assessed through criterion 3. The Cohen's kappa coefficient obtained for this criterion was 0.94. Table IV shows the results of the evaluation of the aspects included in criterion 3 for each of the textbooks.

TABLE IV. Results of criterion 3 evaluating whether the textbooks selected present the electromagnetic field as a single entity. C3.1 refers to, if an electric or magnetic field is observed in one inertial reference frame, both fields are detected in any other reference frame while C3.2 is related to the presentation of the Lorentz transformations and the electromagnetic field as a single entity.

Authors	Criterion 3.1	Criterion 3.2
P. A. Tipler and G. Mosca	NT	NT
J. Walker, R. Resnick, and D. Halliday	NT	NT
D. C. Giancoli	NT	NT
R. W. Chabay and B. A. Sherwood	C	C
R. A. Serway and J. W. Jewett	NT	NT
H. D. Young and R. A. Freedman	NT	NT
R. Knight	C	C
W. Bauer and G. Westfall	NT	NT
D. M. Katz	NT	NT
T. A. Moore	C	C
R. L. Hawkes, J. Iqbal, F. Mansour, M. Milner-Bolotin, and P. J. Williams	C	NT
R. Wolfson	C	NT

Research in physics education has advocated the inclusion of a simplified relativistic approach to electromagnetism in introductory physics courses, in order to establish coherence and encourage the construction of a synthesized view of electric and magnetic fields as complementary facets of the same entity [1]. However, more than half of the textbooks analyzed do not address any of the aspects contained in criterion 3. As far as the analysis of electric and magnetic fields in different reference systems is concerned, fewer than half of the textbooks deal with the subject (criterion 3.1), while field transformations are dealt with by three of the textbooks analyzed (criterion 3.2).

The place within the study of electromagnetism where the aspects mentioned in criterion 3 are addressed changes depending on the text analyzed. For example, Chabay and Sherwood [74] (p. 678), following Biot-Savart's law, explain a "thought experiment" with fictitious students. Jack is sitting in the classroom and has an electrically charged tape and a compass. Since, in his opinion, the tape is at rest, it detects an electric field, but the compass does not deflect. Meanwhile, Jill, who also has a compass in her hands, runs at high speed in front of Jack. Jill detects an electric field, but because in her opinion the charged tape is moving, her compass deflects and detects a magnetic field. Thus, Chabay and Sherwood [74] (p. 678) conclude

> "Up until now we have implied that electric fields and magnetic fields are fundamentally different, but this 'thought experiment' shows that they are in fact closely related."

Furthermore, Wolfson [72] (p. 682) deals with the relative nature of the fields in the chapter on special relativity, after having dealt with Maxwell's laws and electromagnetic waves. To do so, he starts from the example of a charge performing a URM with respect to a certain inertial reference frame and compares the fields that would be detected by an observer located in that frame versus the other who is at rest with respect to the charge. Thus, Wolfson [72] (p. 682) concludes:

> "So electric and magnetic fields aren't absolutes; what one observer sees as a purely electric field another may see as a mix of electric and magnetic fields, and vice versa. You can think of the electric and magnetic field as components of a more fundamental electromagnetic field; how that field breaks out into electric and magnetic fields depends on your frame of reference."

Within the textbooks that address field transformations and present the electromagnetic field as a single entity, we come across two very different approaches to the problem. Knight's textbook [70], for example, has a similar structure to most of the texts analyzed in terms of the order and presentation of the topics and devotes an entire chapter to discuss the electromagnetic field, its transformations and the electromagnetic waves. He starts by comparing the forces on a charged particle in a magnetic field, for observers in different reference frames and assuming that the force is invariant, he concludes [70] (p. 930):

> "Whether a field is seen as "electric" or "magnetic" depends on the motion of the reference frame relative to the sources of the field."

Knight then deduces an approximation for the transformation of the fields at low velocities and introduces the notion of the electromagnetic field [70] (p. 932):

> "We can no longer believe that electric and magnetic fields have a separate, independent existence. Changing from one reference frame to another mixes and rearranges the fields. Different experimenters watching an event will agree on the outcome, such as the deflection of a charged particle, but they will ascribe it to different combinations of fields. Our conclusion is that there is a single electromagnetic field that presents different faces, in terms of E and B, to different viewers."

Furthermore, in Moore's textbook [69], the electromagnetic field is introduced before Maxwell's own laws and after having presented the fields **E** and **B** in situations with constant charges and currents. Moore, after analyzing the electric and magnetic forces on an electron in the neighborhood of a conductor through which current flows for different inertial reference systems and applying the principle of relativity and Lorentz's contraction, concludes [69] (p. 187):

> "…we must recognize that electric and magnetic fields are but two different aspects of an electromagnetic field, whose reality and effects are frame independent, but whose division into electric and magnetic parts is not. The degree to which we interpret a given electromagnetic field as being electric and/or magnetic depends on one's choice of reference frame. For example, the electromagnetic field of a charged particle at rest is purely electric, but is a mixture of electric and magnetic fields in a frame where the particle is moving."

Moore [69] then introduces Lorentz transformations for fields and applies them in order to study the electromagnetic field generated by a charged particle.

VI. DISCUSSION AND IMPLICATIONS

Within the framework of the defined criteria, in this section, we discuss the results of the research questions that examined how the selected textbooks introduced and explained the nature of electromagnetic fields. We start by focusing on the descriptions of Maxwell's equations, then we analyze how the textbooks explain the generation and propagation of electromagnetic waves. We conclude by looking at their explanations as to how the electromagnetic field is presented depending on the frame of reference chosen.

Regarding the introduction of Maxwell's equations and sources of magnetic fields, we found that all the textbooks gave clear explanations of the mathematical relationships between the various terms in Faraday's and Ampère-Maxwell's laws, but most only offered causal interpretations, identifying four possible sources of electromagnetic fields (criteria 1.1 and 1.2, Table II). One could interpret that there are multiple sources of the electromagnetic field, including theoretical constructs such as the flux of the time-varying electric field. However, as it is shown in Sec. III, the field sources are charge and current distributions. The sometimes confusing presentation of Maxwell's equations may play a part in some of the difficulties students display in their understanding of electric and magnetic fields, for instance, when they mix up force with field or do not know how to define the main characteristics of an electric or magnetic field [20,82,83]. If the sources of electromagnetic fields were introduced in line with the epistemological analysis, it would promote easier comprehension and a better understanding of electric and magnetic fields, and it might help prevent students from developing a cause-and-effect interpretation of Faraday's and Ampère-Maxwell's laws. In the case of Faraday's law, for example, a clear account of how a changing current in a solenoid generates both an electric field and a magnetic field (caused by the current) would allow students to visualize how the law describes the mathematical relationship between the circulation of an electric field around a closed curve and the magnetic field flux variation; and without implying a cause-and-effect relationship between the two terms.

As for Ampère–Maxwell's law, we found that most textbooks in the sample (criterion 1.2) attribute the cause of the magnetic field between the plates of a parallel plate capacitor to the displacement current. This interpretation has endured over time and is a possible cause of misunderstandings. Nevertheless, the epistemological analysis of the classical theory of electromagnetism shows that the magnetic field is caused by the conduction currents in the capacitor plates. To avoid confusion, Rosser [75] suggested renaming $\varepsilon_0 d\Phi_E/dt$ the "Maxwell term" and discontinuing the use of "displacement current." Incomplete explanations may complicate the students' ability to understand Maxwell's equations and their application to concrete phenomena by confusing flux changes with field sources [13,14,84,85].

When we analyzed how the textbooks presented the generation and propagation of electromagnetic waves in relation to their sources, we found that most gave suitable explanations about the origin of waves, i.e., that they are caused by time-varying charge or current distributions. Furthermore, two-thirds of the textbooks used arguments based on a supposed mechanism of mutual generation between the time-varying fields to account for the propagation of electromagnetic waves (criteria 2.1 and 2.2, Table III). This kind of explanation may lead students to think that there are multiple sources of the electromagnetic field and promote an incomplete view by considering the electric field and the magnetic field as two independent entities and may have arisen from a causal interpretation of the differential equations in which time was considered an independent variable [65]. Thus, if we apply the differential form of Faraday's law to the fields in an electromagnetic wave, it is reasonable to conclude that the time-varying magnetic field is the cause of the electric field. However, the differential equations do not imply that the change in one phenomenon over time is the cause of the other; they only help affirm, in the case of Faraday's law, for example, that the rate of change of the magnetic field is associated with a spatial variation in the electric field. The causal interpretation between different phenomena connected by a law is based on the explanations of the phenomena and described with semantic rules of correspondence [65], as is the case with Newton's second law, where the differential equation can be interpreted as a typical linear causal relationship in which the force is the cause of the acceleration. The sort of incomplete descriptions of electromagnetic waves that we observed in some of the textbooks could culminate in students finding it hard to assimilate the concepts behind electromagnetic waves and their propagation [86,87].

The inconsistencies detected in our analysis suggest that electromagnetism should be taught with an emphasis on the relationship between the different terms in Maxwell's equations so that students can appreciate that both the electric and magnetic fields in an electromagnetic wave must simultaneously fulfill Maxwell's equations. As such, students could not possibly imagine one field existing without the other and it would be meaningless to ask which is the cause and which is the effect. Each field depends on the other and neither can be considered the source of the other, hence there is a relationship of circular causality between the fields [88]. The fields of an electromagnetic wave constitute a single entity: the electromagnetic field, which is produced by accelerated charges and propagates at the speed of light. We believe that if there were a greater emphasis on these points, along with a suitable description of electromagnetic field sources, students would be able to develop a more accurate mental construct of the generation and propagation of electromagnetic waves by simplifying and clearly separating the two processes. This would set the stage to address the concept of the electromagnetic field and connect both processes through the notion that the fields propagate at a finite velocity.

We have shown that more than half of the textbooks in the study did not include Einstein's contributions to the development of electromagnetic theory (criteria 3.1 and 3.2, Table IV). Yet, an assessment of the asymmetries that appear in electromagnetic phenomena when Maxwell's equations are applied within different inertial frames of reference could help students better understand the relativistic nature of electromagnetic fields and the theory's coherence. Since electromagnetic induction is explained from the perspective of different observers, for example, then it provides the perfect opportunity to explain away these supposed asymmetries in the electromagnetic field and encourage students to think about the explanatory power of electromagnetic theory [1,70,74,89].

An introduction to the Lorentz transformations can be used to resolve the asymmetries and contradictions that arise when electromagnetic theory is applied to different inertial frames of reference and to formally describe the singularity of electromagnetic fields. Considering that the special theory of relativity is covered in more advanced courses, it is hard to imagine the inclusion of the Lorentz transformations in introductory courses on electromagnetism. Nevertheless, we believe it is feasible to introduce a simplified relativistic framework for electromagnetism that is consistent with courses on classical mechanics. The invariance of the force could be used to resolve the contradictions in the predictions made for different observers, which would bestow the Lorentz force a more fundamental status compared to that of the electric and magnetic force [90,91]. This approach could provide a suitable framework for students to understand that the field transformations originate from the need for simple physical situations, by using links between the perspectives of two observers in relative motion to each other [1,70]. Although Galilean transformations do not resolve the asymmetries and contradictions that arise when Maxwell's equations are applied to different inertial frames of reference, their introduction means students can appreciate the inherently relativistic nature of electromagnetism and develop a consistent concept of the electromagnetic field in keeping with current scientific knowledge.

VII. CONCLUSIONS

In this paper, we have drawn on an analysis of 12 textbooks at the introductory physics level. A key feature of the analysis is that it is oriented toward an epistemologically informed approach. Specifically, we have found that in most of the textbooks analyzed, there is a tendency to make cause-effect interpretations between the different terms of Maxwell's equations. Of the 12 books analyzed, 10 (see Table II) explicitly state that Faraday's law implies

TABLE V. Results corresponding to all criteria.

Authors	Criteria 1.1	Criteria 1.2	Criteria 2.1	Criteria 2.2	Criteria 3.1	Criteria 3.2	Criteria satisfied
P. A. Tipler and G. Mosca	C	NC	C	C	NT	NT	3
J. Walker, R. Resnick, and D. Halliday	C	NC	C	NC	NT	NT	2
D. C. Giancoli	C	NC	C	NC	NT	NT	2
R. W. Chabay and B. A. Sherwood	C	C	C	C	C	C	6
R. A. Serway and J. W. Jewett	C	NC	C	NC	NT	NT	2
H. D. Young and R. A. Freedman	C	NC	C	NC	NT	NT	2
R. Knight	C	NC	NC	NC	C	C	3
W. Bauer and G. Westfall	C	NC	C	C	NT	NT	3
D. M. Katz	C	NC	NC	NC	NT	NT	1
T. A. Moore	C	C	C	C	C	C	6
R. L. Hawkes, J. Iqbal, F. Mansour, M. Milner-Bolotin, and P. J. Williams	C	NC	C	NC	C	NT	3
R. Wolfson	C	NC	C	NC	C	NT	3

that a varying magnetic field generates an electric field and that the Ampère-Maxwell law implies that a varying electric field generates a magnetic field, without discussing other more intuitive sources such as electric charges at rest or in motion previously discussed. However, students are expected to recognize that the only sources of any electromagnetic field are charge and current distributions. From the results of Table II, it does not seem that most of the textbooks analyzed to make it easy for students to reach such a conclusion.

Regarding the propagation of electromagnetic waves, 8 of the 12 textbooks analyzed explain it through a mechanism of mutual induction, where a variable electric field generates a variable magnetic field that in turn generates a variable electric field and so on (see Table III). In this way, the fields continuously create each other and travel generating the electromagnetic wave. This ignores the fact that these fields have a common cause and are components of the same entity, the electromagnetic field, and therefore cannot interact with each other [9]. Explanations should facilitate students' understanding of the electromagnetic field as a single entity of dual nature: electric and magnetic, within the theoretical framework of classical electromagnetism.

For the purposes of the introductory physics courses, it is desirable to gain a qualitative knowledge of the relationships between electromagnetic phenomena and the reference system, allowing students to appreciate the coherence of the theory and providing a unified view of the field, which would serve as basic knowledge for subsequent developments in advanced courses. However, the results of the textbook analysis (see Table IV) indicate that less than half of the textbooks explain that, if an electric or magnetic field is observed in an inertial frame of reference, both fields are detected in any other frame of reference, while field transformations are discussed in only three of the textbooks analyzed.

Scientific theories are characterized by their consistency and predictive power. These qualities are widely accepted but are often difficult to convey to physics students. One of the main concerns in teaching the electromagnetic field at an introductory level is that students may use the concept and its laws in a disjointed and often meaningless way when describing electromagnetic phenomena. The results obtained from the textbook analysis conducted in this article suggest that this could be attributed, at least in part, to oversimplified and compartmentalized teaching approaches, which would not help students to build a coherent framework within which to develop Maxwell's laws and the concept of the electromagnetic field.

The inconsistencies in the presentation of the electromagnetic field found in the textbooks in the sample coincide with some of the learning difficulties detected by research in PER [16,82,86]. We agree with van Kampen and De Cook [92] (pp. 5–22): "physics education research concerning the teaching and learning of electricity and magnetism has not had the same impact as that on mechanics." Similar attention to PER results devoted by textbook authors and curriculum designers in other physics topics (e.g., in mechanics) would be desirable in the topic of electromagnetic field sources and their propagation. This attention could help students to a greater understanding.

Textbooks help to reinforce, extend, and consolidate course content to promote thinking and reflection. A well-written and designed textbook can assist student learning. Textbooks that pay attention to the outcomes of PER, such as *Matter and Interactions* [74] and *Six Ideas That Shaped Physics* [69] (see Table V), and whose use in the classroom has been evaluated in relation to the learning achieved by students [93,94] can assist students in their learning. This is not to say that just paying attention to PER results when explaining theory would improve student learning but rather that taking these research findings into account in textbooks is a factor that can improve learning. We hope this study provides some ideas to inspire the development of new teaching materials that are more in line with scientific practice and progress and the current status of classical electromagnetic theory.

ACKNOWLEDGMENTS

The authors would like to thank PEDECIBA (MEC, UdelaR, Uruguay) and express their gratitude for the grant Fisica Nolineal (ID 722) Programa Grupos I + D CSIC 2018 (UdelaR, Uruguay). Part of this research was funded by the Spanish government (MINECO\FEDER PID2019-105172RB-I00).

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